Supplementary material

Shuo Yang, *Student Member, IEEE*

# A. Differences between Existing Literature and This Paper

D

IFFERENCES between the proposed model and those in the existing literature are summarized in Table I.

TABLE I

Differences between Existing Literature and This Paper

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Category | Ref. | Solving method | Consideration of flexible networking devices | | | | | Time scale |
| OLTC | SVC | SOP | ESS | SMART INVERTER |
| Deterministic  method | [5] | Progressive approximation method |  |  |  | √ |  | Discrete-time |
| [11] | Inner hyperbox approximation method |  |  |  | √ |  | Discrete-time |
| [12] | Method based on convex approximation | √ | √ | √ |  |  | Discrete-time |
| [14] | Method based on optimal power flow | √ | √ |  |  |  | Discrete-time |
| [15] | A set of optimization problem |  |  |  |  |  | Discrete-time |
| [17] | Method on TSO and DSO perspective | √ | √ |  |  |  | Discrete-time |
| [18] | Method based on AC optimal power flow | √ | √ |  |  |  | Discrete-time |
| [20] | Interval-constrained, data-driven method | √ | √ | √ |  |  | Discrete-time |
| Uncertainty  method | [6] | Chance constrained optimization model |  |  |  |  |  | Discrete-time |
| [7] | Method based on probabilistic approach | √ |  |  |  |  | Discrete-time |
| [8] | Random Sampling with “beta” PDF |  |  |  |  |  | Discrete-time |
| [9] | Linear robust optimization |  |  |  |  |  | Discrete-time |
| [10] | Two-stage robust optimization | √ | √ |  |  |  | Discrete-time |
| This paper | Chance constrained, BP spline based optimization method | √ | √ | √ | √ | √ | Continuous-time |

# B. Supplementary Descriptions of the Model.

In this part we give supplementary description of the model.

*1) Supplementary description of the distributed PV:* denotes inner polygon approximation method where a secondary constraint is replaced by a set of constraints :

The values of the coefficients are listed in Table ⅠⅠ.

TABLE I

Coefficients of the Linearized Constraints

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 | 0.2679 | -1 |
| 2 | 1 | 1 | -1.366 |
| 3 | 0.2679 | 1 | -1 |
| 4 | -0.2679 | 1 | -1 |
| 5 | -1 | 1 | -1.366 |
| 6 | -1 | 0.2679 | -1 |
| 7 | -1 | -0.2679 | -1 |
| 8 | -1 | -1 | -1.366 |
| 9 | -0.2679 | -1 | -1 |
| 10 | 0.2679 | -1 | -1 |
| 11 | 1 | -1 | -1.366 |
| 12 | 1 | -0.2679 | -1 |

*2) Supplementary description of the SOP:* The complete form of the SOP model is shown in :

where ,,;represent the output active and reactive power at node and node respectively; / represent the minimum/maximum power factory SOP can output;, and represent the maximum allowable apparent power and the minimum and maximum active power output of the SOP respectively.

and are shown in :

*2) Supplementary description of the ESS:* , , and are shown in :

*3) Supplementary description of the FDN network:* The specific formula forare as follows:

where and are the matrix of active and reactive power flowing through branches respectively; is the node-branch incidence matrix including the TDI; is the node-branch incidence matrix excluding the TDI node, where branches away from the node have an element value of in the matrix, towards the node are , and others are ; ,

The specific formula for are as follows:

The specific formula for are as follows:

*3) Supplementary description recasting (12):* For, using the method in [30], it can be transformed into , as the mathematical expectation of Gaussian distribution is zero.

For, it can be further transformed into the following form:

where is the inverse cumulative distribution function of the standard Gaussian distribution, and and is defined as follows:

where is the -th element of

*3) Supplementary description about (18):* In (18), coefficient matrixes and coefficient vectors are defined as follows. Take and as examples:

where and can be found in (12). Other matrixes and vectors can be got similarly. And is defined as follows:

# C. Supplementary Descriptions of Case Studies

*1) Supplementary description of the* *system parameters of the 12-node system:*

TABLE IⅠ

Values of Device Parameters (12-Node System)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name | Values | Name | Values | Name | Values |
|  | 0.04 |  | 0/-2 |  | 0/0.2/0.4/0.6 |
|  | 0.81/0.9604/ |  | 2/0 |  | 0.9025 |
| 1.0404/1.21 |
|  | 2 |  | 0.5/0.5 |  | 1.1025 |
|  | 0.9 |  | 1/1 |  | 0.95 |
|  | 0.01/0.01 |  | 10 |  | 1.05 |
|  | 0.5 |  | 5 |  | 0.98:0.01:1.02 |
|  | 1 |  | 10 |  | 0.01/0.01 |
|  | 2 |  | 1 |  | 0.1 |

TABLE IⅠⅠ

Values of Line Parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Branch number | From | to | Line resistance (Ω/km) | Line reactance (Ω/km) |
| 1 | TDI | 1 | 0.19 | 0.268 |
| 2  3 | TDI | 2  3 | 0.069  0.19 | 0.099  0.268 |
| 2 |
| 4 | 2 | 4 | 0.106 | 0.153 |
| 5 | 2 | 5 | 0.106 | 0.153 |
| 6 | 5 | 6 | 0.106 | 0.153 |
| 7 | 6 | 7 | 0.19 | 0.268 |
| 8 | TDI | 8 | 0.106 | 0.153 |
| 9 | 8 | 9 | 0.19 | 0.268 |

*2) Supplementary description of the system parameters of the 141-bus system:*

TABLE IⅤ

Values of Device Parameters (141-Bus System)

|  |  |  |  |
| --- | --- | --- | --- |
| Name | Values | Name | Values |
|  | 0.04 |  | 0/-2 |
|  | 0.81/0.9216/ |  | 2/0 |
| 1.0816/1.21 |
|  | 2 |  | 0.81 |
|  | 0.85 |  | 1.21 |
|  | 0.01/0.01 |  | 0.95 |
|  | 0.1 |  | 1.05 |
|  | 10 |  | 0.01/0.01 |
|  | 10 |  | 0.1 |

*3) Supplementary description of the system parameters of the 533-bus system:*

TABLE Ⅴ

Values of Device Parameters (533-Bus System)

|  |  |  |  |
| --- | --- | --- | --- |
| Name | Values | Name | Values |
|  | 0.04 |  | 0/-2 |
|  | 0.81/0.9216/ |  | 2/0 |
| 1.0816/1.21 |
|  | 2 |  | 0.9025 |
|  | 0.99995 |  | 1.1025 |
|  | 0.01/0.01 |  | 0.95 |
|  | 0.1 |  | 1.05 |
|  | 10 |  | 0.01/0.01 |
|  | 10 |  | 0.1 |

*3) Supplementary description of the DT model:*

The DT model used in this paper has the following form:

where and dependent variables .

*3) Supplementary description of the base scenario:*

For the 12-node system, the base scenario is set as follows.

Device parameters are set according to Table Ⅱ, and line parameters are set according to Table Ⅲ. As for the load, we consider two kinds of loads, as illustrated in Fig. C-1. Assume that nodes 2, 3, 7, 10 are load1 while the rest load nodes are load2. The total load still follows Fig 8. (a).



**Fig. C-1.** The predicted curve of the load.

For the 141-bus system, the base scenario is set as follows.

Device parameters are set according to Table ⅤⅠ, and line parameters are set according to [32]. In base scenario, assume that nodes 1-70 are load1 while the rest load nodes are load2. However, the maximum instantaneous value is set equal to the load in [32].

TABLE ⅤI

Values of Device Parameters in Base Scenario (141-Bus System)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name | Values | Name | Values | Name | Values |
|  | 0.04 |  | 10 |  | 20 |
|  | 0.81/0.9216/ |  | 10 |  | 10 |
| 1.0816/1.21 |
|  | 2 |  | 0/-2 |  | 0.01/0.01 |
|  | 0.85 |  | 2/0 |  | 0.1 |
|  | 0.01/0.01 |  | 0.5/0.5 |  | 0.81 |
|  | 0.1 |  | 1/1 |  | 1.21 |